Problems for *Wizards, Aliens and Starships*

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Here are a number of sample problems for the book *Wizards, Aliens and Starships: Physics and Math in Fantasy and Science Fiction.* Some of them were beta-tested in a class on the science of science fiction taught at St. Mary's College, but others haven't been. A solution set will be uploaded shortly for these problems.

The mathematical difficulty of a problem is indicated by a number of asterisks placed in front of the problem.

* = no mathematics required
** = only simple arithmetic required
*** = high school algebra and/or trigonometry required
**** = more advanced mathematics required
oe = open-ended problem (requires more than just application of formulas)
pr = class project

Problems for the Introduction

1. (*) What are your favorite fantasy or science fiction books? Do you care about the accuracy of the science in them? What are the factors which make them an enjoyable read?

2. (pr) As you read through this book, think about that science fiction or fantasy story you've always wanted to write, and try to figure out how what you read here will affect the plot.

Problems for Section 1: Potter Physics

Chapter 2

1. (*) Find examples in comic books, movies or TV shows in which the conservation laws of physics are violated, and explain why they are being violated.

2. (*)(Harder than the previous question) Find examples in the media in which they aren't violated.

3. (***) Show that the speed of a point on the Earth's equator is v_e=0.46 km/s, or just about 1,000 mph. Useful data: The radius of the Earth is $r = 6,380$ km, and the rotational period is T=24 hrs = 86,400 s. Show that the speed of a point at latitude ϕ degrees is:

$$v(\phi) = 0.46\,\text{km/s} \times \cos(\phi)$$

4. (***) Harry Potter decides to disapparate from the Burrows in Ottery St. Catchpole (latitude: 51° N) to Hogsmeade village (latitude: 71 ° N). Assuming that momentum is conserved, how fast and in what direction is he moving relative to the ground after he appears? Assume that he disapparates due north and that he was not moving relative to the ground initially.

5. (**) Prof. McGonnagall can turn from a human into a cat and vice-versa. We've already gone through the energy issues involved in going from human to cat in the textbook. Now assume that she turns from a cat into a human, and takes the energy required from the loch near the castle. Assume that the loch is initially at a temperature of 20 C and ends up (after her transformation) completely made of ice at 0 C. What is the required mass and volume of the loch to allow her to do this? Compare this to the actual volume and mass of loch Ness. Remember to include the latent heat of fusion for water in your calculation. No wonder *animagi* are rare!

Chapter 3

1. (*) Think of some more problems involved in lighting large areas using candles only.

2. (**) Look up the luminous efficacy of gaslight as in the Victorian era, and discuss the cost and efficiency of lighting a large castle using gaslight.

Chapter 4

1. (*) List difficulties inherent in human-powered flight, a la the machines of Leonardo da Vinci.

2. (*) The largest dinosaurs were larger than the largest land animals around today. Is this an accident, or are there reasons that this is true? One hypothesis is that the Earth's atmosphere was more oxygen-rich than it is now, meaning that for a given size animal, its metabolic rate could be higher. Why would this encourage large land animals and large fliers?

3. (***) What is the airspeed velocity of an unladen swallow? Could it carry a coconut, either by itself or in tandem?

4. (oe) Discuss whether there is a maximum size (given current technology) for an airplane powered by gasoline-powered engines. Hint: assume that the power of an engine scales linearly with its weight (i.e., double the weight of the engine and you double its power output.) Does the power requirement for flying scale linearly with the weight of the aircraft, or at a higher or lower power?

5. (oe) Is there an upper limit on the size of a land animal determined by its weight? Discuss all of the issues which are involved in this question, including exactly how we mathematically model the shapes of animals.

Problems for Section 2: Space Travel

Chapter 5

1. (*) List all of the problems you can think of facing the design and construction of a flying car.

2. (***) Moore's law states that the number of transistors which can be placed on a computer chip doubles every 2 years. Look up the current

number of transistors available on a chip today and (assuming that chip sizes stay roughly constant) calculate when transistors will reach the size of a single atom.

3. (***) A better limitation of the size of our current transistors comes from quantum mechanics. If the gate of a transistor becomes shorter than the DeBroglie wavelength of an electron, the electron will tend to quantum-mechanically tunnel through the gate – meaning that the gate isn't a gate anymore. The DeBroglie wavelength is given by the formula:

$$\lambda = h/p$$

where p is the electron momentum and h=6.6×10^{-34} Js is Planck's constant. Calculate the average DeBroglie wavelength of an electron at room temperature (300 K) and use that to estimate a) the maximum number of transistors on a chip and b) how long from now we will reach this limit.

4. (**) My current MacBook Pro laptop has an average power use of about 60 W and a 1.4 GHz processor. Estimate the energy cost of a single calculation, and (assuming that the voltage level for a binary "1" is approximately 1 V) estimate the number of electrons transferred for each calculation and the transient current.

5. (***) Look up the formula for the efficiency of the Otto cycle used in car engines as a function of the *compression ratio* of the cylinder. Assuming that most automobiles achieve a compression ratio of about 10, what is the ideal efficiency of the engine? Compare this to the actual average efficiency of about 20%. What is responsible for the large difference here?

6. (oe) Unfortunately, there is no simple way to derive the gas mileage of a car from its engine's efficiency. Look up the following: the effect of air resistance on a car, drive-train efficiency, power requirements for the electrical system, and the effects of starting and stopping versus highway driving. Discuss the physics behind each of these, and how they contribute to the overall gas mileage of the vehicle. Put numbers in where you can.

7. (oe) List all of the things which a human can do in space which a machine can't. (Please keep the discussion "G" rated, thank you very much...) What developments in computer science would be needed to completely replace people in space? Discuss whether you feel that there is a future in the human exploration of space.

Chapter 6

1. (*) Do you want to take a vacation to the ISS? Why or why not?

2. (**) List all infrastructure costs you can think of in getting the space shuttle up into orbit, and estimate them. Is the final value you get in line with the actual cost of about half a billion dollars per mission?

3. (***) Recalculate table 6.1 for u=1,000 m/s, 2,000 m/s and 5,000 m/s. At each ejection speed, what is the mass ratio needed to reach orbital and escape velocity from the Earth?

4. (***)To get to the Moon, a spacecraft needs to get to a final speed of about 11 km/s (i.e., approximately the escape velocity from the Earth.) What fuel mass would be needed to get the space shuttle to the Moon? (Look up the mass of the shuttle online). Compare this to the actual mass of the shuttle's fuel tanks (again, this info is available online.)

5. (***)Assume that we wish to accelerate a rocket to a certain speed, v and then turn the rocket around and decelerate it to zero speed. Show that if the rocket needs a mass ratio of R to reach speed v, it will need an initial mass ratio of R^2 to do the maneuver described above.

6. Let's do a guesstimate problem: you are a NASA engineer planning a manned mission to Mars. Part 1 of the problem: assume that the trip to Mars will take about 6 months to complete. Estimate the average velocity of the spacecraft. State any assumptions you make for this.

7. (**) Part 2 of the problem: Let's oversimplify the trip drastically: we'll assume that to get to Mars and come back, you'll have the spacecraft repeat the maneuver from question 3 *twice*: once, to get to Mars, and second, to come back to Earth. Using the value of u from above, calculate the overall mass ratio and the total fuel mass needed to get

a 10,000 kg payload mass spacecraft to Mars and back. Comment on the difficulties inherent in doing this with chemical rockets.

Chapter 7

1. (*) Find at least three science fiction stories involving space colonies at the L4 or L5 points, and discuss them from a physics perspective.

2. (**) Look up the total budget for running the International Space Station for the last decade. Extrapolate that to the costs of running a station with 10,000 personnel for a decade. Do you think that this is a fair comparison? Why or why not?

3. (***) The space station *Babylon 5* is a cylinder 8 km in length and 1 km in diameter, rotating about its axis once per minute. It has a total mass of 2.5 billion kilograms. What is the speed of a point on its outer hull? Using the relation that kinetic energy is given by:

$$K = \frac{1}{2}mv^2$$

where v is the hull speed and m is the mass, what is the total rotational kinetic energy of the station? If we have a power plant supplying 1 megawatt of energy for energy needs, how long would it take to generate the kinetic energy to rotate the station. Note: Any students who've taken physics will realize that the former discussion is an approximation to the real rotational kinetic energy, which will be less than the amount calculated because of the exact geometry of the station.

Chapter 8

1. (*) It has been said that the space elevator will be built when everyone stops laughing at it. Discuss the idea with your friends, explaining it carefully, and note how many people laugh at the idea versus how many think its interesting and exciting.

2. (oe) One big potential problem with building a space elevator is the amount of space junk in orbit which could collide with it and potentially sever it. Discuss the likelihood of this occurring - put in numbers if you can. As a hint, Bradley Edwards discusses this problem in reference 77.

3. (***) Because the elevator is in tension like a long violin string, it will vibrate like one. It will undergo wave motion where the structure will vibrate with waves moving from the top to bottom and vice versa. The speed of waves on a string is:

$$v = \sqrt{TL/M}$$

where T is the tension in the string, L the length of the string, and M the total mass. (This is only an approximation because the tension and shape of the elevator won't be the same everywhere, very much unlike a violin string.) Estimate the speed of a wave moving along the space elevator, and calculate how long it will take to go from the top to the bottom.

4. (****) Recalculate the shape of the space elevator using the assumption that the internal stress is the same everywhere. Using $h = 2{,}000$ km, what is the value of the internal stress? Hint: See reference 24.

Chapter 9

1. (***) Calculate the eccentricity and trip duration of a spacecraft on a Hohmann transfer orbit from Earth to Venus. The average distance of Venus from the sun is 0.72 AU.

2. (***) Calculate the two Δv's associated with the trip to Venus in the previous problem.

3. (***) If the government is unwilling to fund any projects which take longer than 30 years to complete, estimate the farthest out we can send a spacecraft using a simple elliptical orbit with minimum distance from the sun being Earth's orbit (1 AU). Discuss why we need to help things along with gravitational assists.

Chapter 10

1. (***) At a $1\,g$ acceleration, how long would it take to reach Venus at its closest approach to Earth (about 0.28 AU)? At its farthest? Express your answer in days.

2. (***)Repeat the previous question if the acceleration is "only" equal to 0.1 g.

3. (***)In the previous two problems, how fast is the spacecraft moving at turnover?

4. (***) 1 light-year (LY) is the distance which light travels in 1 year. Show that to a very good approximation, $1g = 1$ LY/year^2. Speed of light: $c = 3 \times 10^8$ m/s.

5. (***) The ejection speed of a gaseous rocket fuel is about equal to the average molecular speed of atoms in the gas. At a given temperature, the lighter the atom used for fuel, the faster the speed will be (this is why hydrogen is a popular fuel.) The formula for the average molecular speed is:
$$v = 157\,\text{m/s} \times \sqrt{T/M}$$
where v is the average molecular speed, T is the absolute temperature (in Kelvin – i.e., degrees above absolute zero), and M the molecular mass in grams per mole (see chapter 14). For molecular hydrogen, $M = 2$. The limitations of a NERVA type space craft is that if we get the engine too hot it will melt. If steel melts at a temperature of about 1,800 K, estimate the maximum ejection speed using molecular hydrogen as a fuel.

Chapter 11

1. (***)Project Daedalus proposed using a fusion-powered interstellar craft to get an unmanned starship to Barnard's Star in a time of approximately 40 years. Barnard's Star is about 6 LY away from us; what mass ratio would the ship need to reach it in this time? Assume a flyby mission only – the ship wouldn't decelerate.

2. (**) Assuming that Barnard's Star had a solar system similar to ours (i.e., a size of about 100 AU), how fast would it take for the Project Daedalus-proposed ship to fly through the system?

3. (***) According to a particular website (http://www.kitsune.addr.com/SF-Conversions/Rifts-Trek-Ships), the Starship *Enterprise* has a total mass of 300,000 metric tons (1 metric ton = 1,000 kg) and can reach 60% of

the speed of light using its sublight engines, which we'll assume work by the annihilation of matter and antimatter. (I expect many letters from fans disputing that particular point...) It claims that the *Enterprise* can travel 5 years without refueling. Let's assume that during that 5 year mission, the *Enterprise* accelerates to this speed and back down to zero ten times. Under these assumptions, what is the total mass of antimatter? Remember that half the fuel is antimatter and half is normal matter.

4. (***) Under the assumption that energy costs in the Federation have dropped to $ 10^{-5} /kW-hr and that antimatter can be produced with a total efficiency $f=10^{-4}$, what is the total cost of the antimatter which the *Enterprise* carries?

Chapter 12

1. (***) In a vain attempt to get the starship *Excelsior* traveling faster than the speed of light, her captain orders the crew to accelerate the ship at 1 g for 1.5 years (as measured by clocks on Earth), assuming that it will be traveling at a speed of 1.5 times the speed of light at the end of this time. What is the true speed of the ship, using formula 12.7? Express the answer as some number times c (the speed of light); keep 3 decimal places of accuracy. The program Microsoft Excel has the tanh function built in.

2. (***) How much time has elapsed on clocks on the ship?

3. (***) What is the mass ratio required for this experiment, assuming matter-antimatter annihilation as a fuel for the starship? What is it if the ship wants to slow back down to a halt?

Chapter 13

1. (***) It has been shown that for a wormhole to be stable, it must be threaded by *exotic material* to keep it open. Exotic material is material which has a tension greater than its energy density as measured in some reference frame. The maximum tension which one can stretch a material is less than Young's modulus for most things; look up Young's modulus for stainless steel and compare it to the energy density of steel

(i.e., its density multiplied by c^2). Discuss why such "exotic material" will probably be hard to come by.

2. (***) Let τ_0 represent the tension and ρ_0 represent the energy density of some piece of matter in its rest frame. Under the Lorentz transformations, an observer moving at speed v with respect to that rest frame will measure the energy density to be:

$$\rho = \gamma^2 \left(\rho_0 - v^2 \tau_0 \right) \tag{1}$$

(I am working in units where c=1). Show that in some reference frame exotic matter has a negative energy density, and calculate the minimum velocity in which this is true to the observer moving through it. Explain why this is highly unusual.

Problems for Section 3: Worlds and Aliens

Because of the interrelation of many of the themes in this section, I haven't divided the problems by chapter.

1. (**) If we want to communicate with aliens on a timescale commensurate with a typical human lifespan, then the nearest aliens should be no more than about 20 to 30 light years away from us. Under this assumption, estimate the total number of alien races in the galaxy.

2. (**) The mathematician John von Neumann proposed a scenario whereby the galaxy could be settled and explored in a short (by cosmic standards) time, even if we are constrained to travel at speeds much slower than light. Assume that Earth is able to establish a colony on a planet circling another star in 100,000 years; after another 100,000 years, Earth and its new colony each establish new colonies, and so on. If there are 10,000,000 planets capable of supporting life in the galaxy, how long will it take under this scenario for the galaxy to be completely colonized?

3. (**) The planet Vulcan from *Star Trek* was said to circle the star 40 Eridani A, Look up this star's mass and luminosity; assuming that Vulcan circles the star at a distance so that the radiant flux it receives is the same as the Earth gets from our sun, calculate the distance and period of Vulcan's orbit.

4. (**) Assume that the Vulcans have a radio transmitter capable of transmitting a signal with a power of 10^{11} W. Find the distance to 40 Eridani A; assuming that the signal is broadcast in all directions, calculate the power received by a radio telescope on Earth with a 300 m diameter circular receiver.

5. (pr) Choose a main-sequence star and design a habitable planet circling it. Decide on a size, average orbital distance, eccentricity, whether or not it has a moon, and any other factors you can think of to make the world seem a fleshed-out place.

6. (***)The Earth's population doubles roughly every 20 years. Let's assume that a bunch of invading aliens have a similar problem and want to control it by sending us their excess population Roughly how many would they need to ship to us every day to keep the population under control?

7. (***) Let's say that they want to invade Earth to steal all of our resources. Is this economically sensible? Instead of this they could mine the interior of their own planet for resources. Compare the change in gravitational potential energy of moving 1 kg of material from the center of the Earth to its surface, to the change in potential energy of moving 1 kg of material from infinitely far from the sun to the orbit of the Earth. (Note: you will need to look up the formula for the gravitational potential of a sphere.)

8. (**) Let's say that the invading aliens want to enslave us to do their bidding in their underground dilithium mines. A human can output power at a maximum rate of about 100 W. Estimate the power output of a tractor and see if this makes any sense.

Problems for Section 4: Year Googol

Chapter 17

1. (**) It is a truism that a demand for a quantity which increases exponentially over time will rapidly exceed any possible source. Consider the following well-known story: according to legend, the game of chess was invented by a vizier of a small Indian kingdom to please his Rajah.

The Rajah in delight offered the vizier any gift: the vizier's request was deceptively simple. He asked that the Rajah take one grain of rice and put it on the first square, then put two on the second, and four on the third, eight on the fourth, and so on, doubling the number of the previous square on the next. Assuming that there are 32 grains of rice per gram: at how many squares will the total weight of rice exceed 1 kilogram? 1,000 kilograms? The current world production per year is about 300 million metric tons (300 billion kilograms) – how many squares until the total amount of rice on the chess board exceeds this?

2. (oe) Use the results of the last discussion to consider the dilemma the world finds itself in right now, with exponentially increasing demands on energy but only a finite supply of fossil fuels.

Chapter 18

1. (oe) Larry Niven once wrote that we both understand too much and too little about terraforming planets to be able to discuss it intelligently. Discuss the meaning of this comment.

Chapter 19

1. (***) Assume that we take Jupiter apart and reconstruct it as a Dyson shell centered at 1 AU around the sun. Estimate the total energy it would take to tear Jupiter apart (Hint: look at its gravitational self-energy). How long does it take the sun to generate that much energy?

2. (***) Same problem as above: Estimate the total energy it takes to move a 1-Jupiter-mass object from a distance of 5 AU from the sun to 1 AU from the sun. Again, how long does it take for the sun to generate that much energy?

3. (***) If we reassemble Jupiter into a Dyson shell made of material with an average density equal to water, how thick will the shell be?

4. (***) Repeat the calculation of the last question for a Ringworld of radius 1 AU from the sun.

5. (***) Show that the Ringworld has to spin at a speed of about 800 km/s to generate 1 g acceleration at its surface. How does this compare to the escape velocity of an object 1 AU away from the sun?

Chapter 20

1. (***) Using the data below, estimate when the world will become a Type-I civilization according to the Kardashev scale:

Year	Energy production Exajoules/year	Terawatts	Kardashev rating
1900	21	0.67	0.58
1970	190	6	0.67
1985	290	9.2	0.69
1995	360	12	0.7
2000	420	13	0.71
2004	440	14	0.71
2010	510	16	0.72

Note: 1 exajoule = 10^{18} J.

Also, $K = \frac{\log_{10} P - 6}{10}$

K = Kardashev number, P = world power usage (Watts).

2. (***) The total power output of our sun is about 3.8×10^{26} W. Show that a type-II Kardashev civilization uses about the same amount of power as output by the sun.

3. (***) Show that if the total power used by an industrial civilization doubles, its Kardashev rating increases by 0.03.

4. (***) Estimate the total power output of all of the stars in the galaxy. The total number of stars in our galaxy is approximately 10^{11}. Assuming the exponential growth curve implied by the table above, how long will it be before our civilization uses all of the power of all of the stars in the galaxy?

Chapter 21

1. (*) What is your favorite "end of the world" or story set at the end of time? How realistic is it by scientific standards?

2. (***) The quantum mechanical wavelength of a particle with momentum p is given by the formula

$$\lambda = h/p$$

If the particle is a photon, it will have energy $E = pc = hc/\lambda$. One way of stating the Heisenberg uncertainty principle is that it is impossible to confine a particle in a space much smaller than its wavelength. This is the idea behind black hole radiation: if the event horizon has radius R, it will "leak" light which has wavelengths much longer than R. Using this, calculate the "characteristic" energy of photons leaked away by black holes which have 1 solar mass and 1,000,000 solar masses. Use equation 21.6, but be careful to convert kilometers to meters.

3. (***) We can estimate the black hole temperature from the characteristic photon energy radiated away by it. The estimate is that

$$kT \approx E$$

where $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, and T is measured from absolute zero temperature. Estimate the temperature of a 1 solar mass black hole. Show that your estimate of the temperature is approximately the same as equation 21.7. As a black hole evaporates from Hawking radiation, does its temperature increase or decrease?

4. (pr) Go write that story you had in mind at the beginning of the problems!

But Wizards, Aliens and Starships is not a textbook, nor is it written as such. The entertaining opening section on the physics underlying the magic in fantasy writing investigates some common dilemmas. What to do with the excess mass when the character Draco Malfoy, in J. K. Rowling's Harry Potter books, is turned into a ferret? Ideas include the space elevator, as in Arthur C. Clarke's The Fountains of Paradise, and the reality of space colonies depicted in novels such as George O. Smith's The Complete Venus Equilateral. Adler concludes this section with a discussion about the likelihood of aliens existing, the possibility of time travel and whether humans could live on other planets. Problems from Another Time. Conference Calendar. Guidelines for Convergence Authors. In part III, three chapters totaling about 50 pages, professor Adler looks at issues connected with alien life forms, including what a planet supporting alien life would have to be like, the history and future of the search for alien life, and the issues inherent in actually communicating with aliens (and how science fiction authors have treated these issues). Wizards vs Aliens is a British science fantasy television programme produced by BBC Cymru Wales and FremantleMedia Enterprises for CBBC and created by Russell T Davies and Phil Ford. The series focuses on the exploits of 16-year-old wizard Tom Clarke (Scott Haran) and his scientifically-gifted best friend Benny Sherwood (Percelle Ascott) in their fight against the Nekross, a magic-consuming alien race who have arrived on Earth with the intention of hunting down wizards and feasting on their magical Wizards, Aliens, and Starships will speak to anyone wanting to know about the correct--and incorrect--science of science fiction and fantasy. ...more. Get A Copy. Amazon. The big problem here is the efficiency and fuel requirements of chemical rockets – the only form of interplanetary propulsion likely to be feasible for carrying humans to Mars in the next several decades. A fairly simple equation, the "rocket equation", is explained. It demonstrates that the serious hang-up is the rather large amount of fuel that has to be carried in proportion to payload. "Wizards, Aliens, and Starships rigorously applies the principles of physics to concepts, plot devices, and other features of science fiction and fantasy books, films, and television series. Readers who follow Adler's carefully developed analyses will learn a great deal about familiar science fiction tropes, physics, and how scientists think about the world. An exceptional book."―A. Bowdoin Van Riper, author of Science in Popular Culture. From the Inside Flap. "To only call Wizards, Aliens, and Starships engaging would be a real understatement--it is a delightful, funny, an

Wizards, Aliens, and Starships will speak to anyone wanting to know about the correct--and incorrect--science of science fiction and fantasy. Year: 2014. I've also included solutions and hints for the problems. This book cannot be used to replace a physics textbook, but it could be used in a specialized course. 1.4 the mathematics and physics you need. Wizards, Aliens, and Starshipswill speak to anyone wanting to know about the correct--and incorrect--science of science fiction and fantasy. eISBN: 978-1-4008-4836-2. Subjects: Physics, Mathematics. Compounding the problem, there is a plethora of recent books on the subject, and almost anything I can say about the subject has already been said by those authors, and better. Faster-than-light (FTL) travel is a key component of many science fiction stories, for obvious reasons: traveling to the stars Cite this Item. III Worlds and aliens. Chapter fourteen designing a habitable planet. Chapter fourteen designing a habitable planet. (pp. 217-241). Wizards vs Aliens is a British science fantasy television programme produced by BBC Cymru Wales and FremantleMedia Enterprises for CBBC and created by Russell T Davies and Phil Ford. The series focuses on the exploits of 16-year-old wizard Tom Clarke (Scott Haran) and his scientifically-gifted best friend Benny Sherwood (Percelle Ascott) in their fight against the Nekross, a magic-consuming alien race who have arrived on Earth with the intention of hunting down wizards and feasting on their magical