DEFINING VISIONS OF HIGH-QUALITY MATHEMATICS INSTRUCTION

Charles Munter
Vanderbilt University
c.munter@vanderbilt.edu

By synthesizing what has been learned with regard to critical dimensions of mathematics classroom teaching and learning, and investigating the ways teachers and other school personnel characterize high-quality mathematics instruction, this study defines the notion of 'instructional vision' and provides an initial categorization scheme. Motivating this work is the need to reliably document change in participants' instructional visions within an ongoing study of the institutional setting of mathematics teaching, in which we hypothesize that improvement in teachers' instructional practices and student achievement will be greater in schools where teachers and instructional leaders have a shared vision for high-quality mathematics instruction.

Background

The Middle School Mathematics and the Institutional Setting of Teaching (MIST) research team is working for four years with four urban school districts serving ethnically and economically diverse populations as 'co-designers' of support structures and strategies for meeting ambitious goals for reforming mathematics instruction at a district level. All four of our participating districts have recently formulated and begun implementing comprehensive initiatives for improving the teaching and learning of middle-school mathematics, including the adoption of mathematics reform curricula and the provision of professional development aimed at developing instructional practices in which teachers place students' reasoning at the center of their instructional decision making.

Our aim is to investigate, test, and refine a set of conjectures and formally test a set of hypotheses about support structures that potentially enhance the impact of professional development on middle school mathematics teachers' instructional practices and student achievement. Our conjectures and hypotheses pertain to seven sets of support structures: 1) teachers' professional networks; 2) shared vision for high quality mathematics instruction across teachers and instructional leaders; 3) quality of instructional leadership; relationships of 4) accountability and 5) assistance between teachers and instructional leaders (including principals, assistant principals, department chairs and mathematics coaches) and among teachers; 6) alignment across district units with respect to high-quality mathematics instruction; and 7) particular supports for providing equitable learning opportunities to all students.

It is the second of these hypotheses that motivates the work represented in this paper: Improvement in teachers' instructional practices and student achievement will be greater in schools where teachers and instructional leaders have a shared vision for high-quality mathematics instruction. Eventually, the goal is to establish a means of tracking shifts toward both increased sophistication and 'sharedness' in individual leaders' and teachers' instructional visions. However, in order to determine the extent to which groups of individuals *share* an instructional vision, we must first be able to reliably assess accounts of personal visions of high-quality mathematics instruction. With this paper I define *visions of high-quality mathematics* and describe the initial categorization scheme resulting from preliminary data analysis.

Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.). (2009). Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Atlanta, GA: Georgia State University.

Theoretical Perspectives

Professional Vision

Charles Goodwin (1994) presented a comparative analysis of practices in two professional settings, an archeological field school excavation and the 1992 California trial of four policemen charged with beating Rodney King. He argued that both the senior archeologist and the defense attorneys (with the help of an "expert witness"—an LAPD sergeant who was not present for the alleged beating) utilized particular complex discursive and representational practices to build and contest professional vision, which he defined as "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606).

Sherin (2001) extended the idea of professional vision to her work in documenting the evolution of one mathematics teacher's perspective of classroom events. Her teacher, David Louis, had been teaching for five years at the time that Sherin and her colleague began observing and videotaping his classroom and meeting weekly with him to watch excerpts of those video recordings. During the most recent year-and-a-half, Mr. Louis had attempted to change his instructional approach to one of supporting the development of a community of learners (Brown & Campione, 1996; Rogoff, Matusov, & White, 1996). In addition to their weekly video viewing sessions, Mr. Louis and the researchers also participated monthly in a video club with David's colleagues. Over the course of her 4-year collaboration with Mr. Louis, Sherin documented how his interpretation of classroom events captured on video from his classroom changed from a focus on his own pedagogical actions (i.e., what he should have done differently) to one on student ideas and the nature of mathematical discussions (i.e., accounting for what had actually transpired in classroom events of interest). Adapting Goodwin's notion, Sherin suggested that this marked a shift in his professional vision—a "new interpretation strategy" (p. 90) focused more on the aspects of classroom activity to which researchers rather than teachers typically attend.

However, in his conception of professional vision, Goodwin was much less concerned with how individual actors had come to the point of being able to enact the practices of a professional vision than he was in examining how the enactment of a professional vision is accomplished. Although he recognized that the practices must be learned, his conception of professional vision was much more collectively and historically oriented, arguing that "the ability to see relevant entities is not lodged in the individual mind, but instead within a community of competent practitioners" (p. 626). In her account of Mr. Louis's interpretations of classroom events, Sherin explicitly made the leap from archaeology to mathematics teaching, but in doing so mapped Goodwin's (collective) professional vision within archaeology onto (individual) ways of interpreting events in the mathematics classroom.

Though Sherin's analysis of Mr. Louis's shift in perspective might not have adhered faithfully to Goodwin's conception of professional vision, there is much to be learned from her account. Sherin adapted Goodwin's notion of "professional vision" to describe one individual's way of seeing and interpreting classroom events. Across time, she documented which aspects of the classroom the teacher emphasized as being important with respect to mathematics instruction and learning, and the rationale behind his choices. It is this perspective on the classroom that I refer to as simply a "vision"—specifically, a "vision of high-quality mathematics instruction" (for which I will use "instructional vision" synonymously). Just as Sherin was able document an evolution in Mr. Louis's way of seeing and interpreting classroom events, the motivation for the work reported in this paper arose from a need to reliably document change in the visions of high-quality mathematics instruction among our study participants (including mathematics teachers, principals, mathematics coaches, and district leaders) to provide a means for determining the extent to which groups of participants move toward a shared instructional vision. To achieve this goal, it is therefore necessary to build a framework for considering the ways teachers and other participants characterize high-quality mathematics instruction.

Dimensions of High-Quality Mathematics Instruction

A considerable body of literature provides insights into one or more important aspects of mathematics teaching and learning, often investigated and reported as discrete elements of the practice. However, few attempts have been made to glean a coherent set of distinct aspects that adequately delineate crucial dimensions of the practice. In the following paragraphs, I summarize three such attempts, namely those of Franke, Kazemi, and Battey (2007); Carpenter and Lehrer (1999); and Hiebert and colleagues (1997). By examining how the various summaries fit together, my intention is to provide an initial frame for the analyses presented later in the paper.

Franke, Kazemi, and Battey (2007) described three features of mathematics classroom practice they viewed as most central: 1) creating and shaping mathematical *classroom discourse*; 2) establishing classroom *norms* for doing and learning mathematics; and 3) building *relationships* with and among students that support participation in the mathematical work of the classroom. The authors further detailed specific aspects of each feature. For example, with respect to classroom discourse, Franke and colleagues stressed four core ideas and practices, including revoicing student thinking (O'Connor & Michaels, 1993) to highlight particular mathematical ideas, to introduce mathematics vocabulary or to position students in relation to each other and their arguments; employing tasks that provide for multiple strategies and rich discussion; identifying and building on the resources English language learners bring to mathematical discussion; and encouraging students to interrogate meaning (Rosebery, Warren, Ogonowski, & Ballenger, 2005) behind mathematical assumptions and ideas, which contributes to developing classroom norms around questioning and challenging. Regarding classroom norms, the authors stressed the importance of distinguishing between social and sociomathematical norms (Yackel & Cobb, 1996), and attending to the consequences such norms have for student learning and defining what it means to 'do mathematics.' Lastly, the authors described the importance of teachers building relationships with students in terms of understanding children's thinking, and also in ways that lead to opportunities for participation, "which requires getting to know students' identities, histories and cultural and school experiences, all in relation to the mathematical work" (p. 243).

Carpenter and Lehrer (1999) described three dimensions of instruction, the examination of which they viewed as critical for enabling students to engage in mental activities necessary for learning mathematics with understanding: *mathematical tasks*, *tools* and *normative practices*. First, the authors suggested that through task sequencing that is based on children's thinking rather than mathematical structure, the learning of concepts and skills can be integrated. Important in this vein is that tasks be viewed as problems to be solved, not exercises to be completed, and that they be couched in meaningful contexts. Secondly, the authors suggested that tools, such as paper and pencil, manipulatives, calculators, computers and symbols, be used to represent mathematical ideas and problem situations. They argued that "connections with representational forms that have intuitive meaning for students can greatly help students give meaning to symbolic procedures" (p. 25). By considering the use of such tools, which can be introduced by either students or the teacher, students begin to abstract the mathematical ideas behind their manipulations, so that they gradually no longer need the physical representations. But the authors also argued that it is not the tasks and tools alone that will support learning with understanding. Lastly, Carpenter and Lehrer pointed to the role of classroom normative practices, which influence the use and interpretation of tasks and tools and "govern the nature of the arguments that students and teachers use to justify mathematical conjectures and conclusions" (p. 26). A key norm the authors highlighted is that students be expected to regularly discuss alternative strategies and why they work. This practice, they argued, will not only motivate the kinds of reflective mental activity previously described as students come to participate in what has been established as common classroom practice, it will also provide opportunities to make relationships explicit as the class examines how various methods are alike and different.

Predating the work described above was Hiebert et al.'s (1997) book, *Making Sense: Teaching and Learning Mathematics with Understanding*. Based on research conducted by the eight authors in various mathematics classrooms, Hiebert and colleagues identified and devoted a chapter to each of five "dimensions" of mathematics classroom instruction and activity: 1) the nature of classroom tasks; 2) the role of the teacher; 3) the social culture of the classroom; 4) mathematical tools as learning supports; and 5) equity and accessibility. Within each of these dimensions, the authors discussed essential "core features," necessary for supporting students' understanding of mathematics. Like the authors whose work is discussed above, Hiebert and colleagues attempted to describe a set of features of mathematics classroom instruction they viewed as critical for providing opportunities to learn mathematics with understanding—the dimensions that 'matter.' Additionally, the authors viewed their framework as potentially meaningful to those engaged in the practice of mathematics instruction, suggesting that it could be "used by teachers to reflect on their own practice, and to think about how their practice might change" (p. 3).

Hiebert et al. (1997)	Carpenter & Lehrer (1999)	Franke et al. (2007)
Nature of Classroom Tasks	Tasks or activities students engage in and the problems they solve	
Role of the Teacher		
Social Culture of the Classroom	Classroom normative practices	Establishing norms for doing and learning mathematics
Mathematical Tools as Learning Supports	Tools that represent mathematical ideas and problem situations	
Equity and Accessibility		Building relationships for doing and learning mathematics
		Supporting discourse for doing and learning mathematics

Figure 1. Summary of central aspects of mathematics instruction identified in three works.

In Figure 1 I list the critical dimensions of mathematics classroom instruction identified in the work summarized above, mapping those identified by Carpenter and Lehrer and those of Franke et al. onto the dimensions described by Hiebert and colleagues. The 'gaps' in these lists should not be interpreted as omissions; they are typically a consequence of arrangement and classification choices. Since all of the authors acknowledged a systemic relationship among their dimensions, it is not surprising that in any one of these classification choices, dimensions identified as central by the other authors can be found. Two particular instances worth noting in this regard are Hiebert et al's "role of the teacher" and Franke et al.'s "supporting discourse for doing and learning mathematics." Much of what Carpenter, Franke and their co-authors wrote about pertained very much to the role they envisioned a teacher playing. For example, both teams described the importance of the teacher's influence on the establishment of classroom norms. Likewise, Franke et al.'s discourse dimension was represented in multiple places throughout the others' summaries as they discussed the importance and role of communicating about mathematics in the classroom. Although it is to some extent a matter of reorganizing and renaming, I will argue below that the labels on the dimensions are meaningful in that they can represent points of view, or ways of seeing and valuing aspects of a mathematics classroom.

Research Questions

As stated above, my immediate goal was to build a coding scheme for assessing participants' visions of high-quality mathematics instruction. Therefore, my question regarded the ways teachers, principals, and others characterize high-quality mathematics instruction. In particular, which aspects of mathematics classroom instruction do they choose to highlight? To what extent do practitioners' characterizations of classroom instruction map onto the critical "dimensions" described in the literature?

Methodology

Participating school districts were purposively sampled to represent districts with ambitious goals for mathematics instruction reform to meet the needs of diverse populations. While differences exist among the strategies the districts are attempting to implement for accomplishing their goals, in general, all four districts are working to support mathematics instruction in every classroom that emphasizes rigorous tasks, problem-solving and sense-making, productive discourse, fair and credible evaluations, and clear, high-level expectations for all students.

In each annual data collection we document aspects of the institutional settings in which our participants work, the instructional practices and mathematics content knowledge for teaching of approximately 30 middle school mathematics teachers per district, and the extent to which structures such as those listed above have been established to support the ongoing improvement of mathematics teaching in 6-10 representative middle schools per district. Annual data sources are varied, but include 45-90 minute interviews with each participating teacher, principal, mathematics coach and district leader on issues related to the institutional settings in which they work, as well as their vision of high-quality mathematics instruction.

The data analyzed for this paper come from the interviews conducted in year one (January 2008) with middle school mathematics teachers, principals, and, in districts that employ them, mathematics coaches. I examined transcripts from every participant (teachers, principals and coaches) at each of eight schools (two from each district). These schools were theoretically sampled (Strauss & Corbin, 1998) to provide wide variation in the ways participants talked about mathematics instruction, as indicated in case summaries written for each school. The interviews were conducted and audio-recorded by members of the project team and later transcribed. As is the case with 'unstructured interviews,' (Burgess, 1984), they followed a set of guiding questions (that were customized for each district), but were conducted flexibly, intended to afford opportunities for conversations rather than a rigid sequence of questions.

The interviews probed at a number of issues related to mathematics instruction and the institutional setting in which participants work, including participants' understanding of the district's plans for improving mathematics instruction, their vision of high-quality instructional leadership, their informal professional networks, professional development activities in which they participate, the people to whom they are accountable, the sources of assistance on which they draw, and the curriculum materials they use in the classroom. An additional purpose of the interviews—and the focus of my analyses in this paper—was to document teachers', principals', and coaches' visions of high-quality mathematics instruction. Specifically, we asked participants the following question: "If you were asked to observe another teacher's math classroom, what would you look for to decide whether the mathematics instruction is high quality?" Depending on the participant's response, we asked, "Why do you think it is important to use/do _____ in a math classroom? Is there anything else you would look for? If so, what? Why?"

The purpose of this particular question was twofold. First, it was an attempt to circumvent the say-do problem (Gougen & Linde, 1993), a well-known obstacle in the social sciences in that self-reporting typically does not yield reliable data concerning participants' own practices. Thus, in asking our participants to imagine and talk about the activities in a classroom of some hypothetical other, our thinking was that we could release the participant from some of the pressures of accurately describing his/her own classroom and practices, and tendencies to foreground more socially desirable aspects of classroom instruction to the exclusion of those perceived as less desirable . Secondly, and more importantly, in asking teachers (and principals and coaches) to place themselves in the role of observer, we hoped to ascertain aspects of the lens with which each participant would actually view a mathematics classroom, or the way they interpret classroom events (Sherin, 2001). That is, we could interpret their responses to mean, "this is what matters in a mathematics classroom"—the aspects of the classroom on which they focus to determine the quality of instruction. This in turn would enable us to not only establish the kinds of things they might attend to when observing a mathematics classroom (e.g., what the teacher does, what the students are doing, the nature of the mathematical tasks, the nature of classroom discourse, etc.), but also to attribute some measure of depth or sophistication to their criteria.

Focusing in particular on the portion of the interviews including the question and probes mentioned above, I collected more than 200 statements from 54 of 222 participants (8 principals, 41 teachers and 5 mathematics coaches). Following Strauss & Corbin's (1998) open coding technique, I classified these statements (or concepts) into categories based on shared properties, such as whose behavior the statement pertained to (e.g., teacher, students, or both) or which aspect of the learning environment was emphasized (e.g., nature of classroom tasks, structure of lessons, etc.).

My initial classification was guided by a provisional list of codes (Miles & Huberman, 1994) drawn from Hiebert and colleagues' (1997) identification of essential dimensions of mathematics classroom instruction discussed above. I employed these dimensions and their accompanying core features as an initial framework for categorizing mathematics teachers' and instructional leaders' descriptions of what they would look for in a classroom to determine whether what they observed was high-quality instruction. Hiebert and colleagues' framework represented a reasonable starting point for my purposes, since the authors viewed it as a tool that could be used by those engaged in the practice of mathematics instruction for reflection and change. Thus, in initially adopting these authors perspective, I was not merely imposing a priori a researcher's tool on practitioners' views, but attempting to combine etic and emic accounts to create tools in which both communities find relevance and meaning.

Results

Approximately half of the lines of talk I collected from interview transcript resembled the kinds of ideas expressed in the dimensions proposed by Hiebert et al. Many participants commented on the types of problems they would hope to see students working on (the nature of classroom tasks), what they thought the teacher should be doing (the role of the teacher), or how students would be interacting with other students and the teacher (the social culture of the classroom). Approximately five participants commented on the need to differentiate instruction based on students' individual needs (equity and accessibility), and two participants' remarks pertained to the importance of technology or means of representation in the classroom (mathematical tools as learning supports). Since my primary goal was to describe participants' visions of practice, I decided to drop the category pertaining to mathematical tools because it accounted for so few of the participants' responses, and retain the other four dimensions proposed by Hiebert *et al*. But a considerable number of responses were left unaccounted for. Therefore, I sorted the remaining statements into groups that appeared to share a focus. One set of concepts pertained to student and teacher talk (i.e., classroom discourse—a dimension identified in the framework of Franke *et al.*), one to lesson structure, another to assessment, and another (the largest) to student engagement.

In summary, in order to establish a means for describing participants' instructional visions, I have relied on both our interview data and previous work in identifying important aspects of mathematics classroom practice to identify categories to which teachers' and leaders' instructional visions might pertain. This analysis resulted in eight categories or, in Hiebert and colleagues' language, "dimensions": 1) the role of the teacher; 2) classroom discourse; 3) the organization/purpose of activity (i.e., student engagement); 4) social culture and norms; 5) the nature of classroom tasks; 6) role of student thinking; 7) lesson structure; and 8) equity and accessibility.

Discussion

With this paper I have attempted to define a construct important to the ongoing MIST project, that of instructional *visions*. In the spirit of Simon and Tzur's (1999) efforts to generate "accounts of mathematics teachers' practice," my aim was to categorize and understand teachers' visions of high-quality mathematics instruction "in a way that accounts for aspects of practice that are of theoretical importance to the communities of mathematics education researchers and teacher educators" (p. 254). Thus, guiding my analysis was this question: What is the minimum number of dimensions of classroom activity and instructional practice that makes a difference? This question represents a two-pronged endeavor. One the one hand, I wanted my final categories to reflect what previous researchers have identified as important aspects of mathematics classroom instruction. On the other, I needed the final categories to sufficiently and meaningfully account for patterns I perceived in our data.

Along each of the eight dimensions listed above, we have elaborated a conjectured trajectory for participants' instructional visions in terms of depth and sophistication of their descriptions (the presentation and discussion of which is beyond the scope of this paper). Over the 4-year duration of our research team's work in school districts, we will use (subsequent refinements of) these trajectories to document shifts in individuals' visions of high-quality mathematics instruction, and the extent to which members of various district units move toward a shared instructional vision.

References

Brown, A. L., & Campione, J. C. (1996). Guided discovery in a community of learners. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 229 –270). Cambridge, MA: The MIT Press.

Burgess, R. G. (1984). Methods of field research 2: Interviews as conversations. *In the field* (pp. 101-122). London: George Allen & Unwin.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19-32). Mahwah, NJ: Lawrence Erlbaum Associates.

Franke, M. L., Kazemi, E., & Battey, D. (2007) Mathematics teaching and classroom practice. In F. K. Lester, (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Reston, VA: NCTM.

Goodwin, C. (1994). Professional Vision. *American Anthropologist, 96,* 606-633.

Gougen, J. A., & Linde, C. (1993). Techniques for Requirements Elicitation. *Proceedings of the IEEE International Symposium on Requirements Engineering.* January 4-6, 1993, San Diego, California, IEEE Computer Society Press, Los Alamitos, California, 152-164.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Thousand Oaks, CA: Sage.

O'Connor, M. C., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly, 24,* 318-335.

Rogoff, B., Matusov, E., & White, C. (1996). Models of teaching and learning: Participation in a community of learners. In D.R. Olson & N. Torrance (Eds). (1996). *The Handbook of education and human development*. (pp.388-414). Cambridge, MA: Blackwell Publishers, Inc.

Rosebery, A., Warren, B., Ogonowski, M., & Ballenger, C. (2005). The generative potential of students' everyday knowledge in learning science. In T. Carpenter & T. Romberg (Eds.), *Understanding matters: Improving student learning in mathematics and science* (pp. 55-80). Mahwah, NJ: Erlbaum.

Sherin, M. G. (2001). Developing a professional vision of classroom events. In T. Wood, B. Scott Nelson, & J. Warfield (Eds.) *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 75-93). Mahwah, NJ: Erlbaum.

Simon, M., & Tzur, R. (1999). Explicating the teacher's perspective from the researchers' perspectives: Generating accounts of mathematics teachers' practice. *Journal for Research in Mathematics Education, 30,* 252-264.

Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction, 25*(2), 161-217.
Strauss, A., & Corbin, J. (1998). Basics of qualitative research: Techniques and procedures for developing grounded theory (2nd ed.) Thousand Oaks, CA: Sage.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458-477.

Detailed vision of high-quality mathematics instruction. A second aspect of a coherent instructional system is a detailed vision of high-quality instruction that specifies concrete instructional practices that have the potential to lead to the attainment of the learning goals. This vision articulates the goals for teachersâ€™ learning. Consistent with current research on teacher professional development, we have found it important that the guiding vision of instruction specifies a relatively small set of high-leverage instructional practices that are learnable in the context of high-quality prof Wisconsin schools have the responsibility to ensure that all students receive engaging, high-quality mathematical teaching aligned to grade-level standards. Mathematics teaching involves not only helping students develop mathematical skills but also empowering students to see themselves as being doers of mathematics.Â Defining universal, selected, and intensive practices and supports for mathematics instruction at the school level. The foundation of a schoolâ€™s mathematics instruction is the universal level, where all learners receive strong, high-quality teaching.Â A systemic and systematic framework creates a vision for quality mathematics instruction that is shared by everyone in a school district. An instructional framework Keywords. Mathematics Education Classroom Teacher Assessment Practice Assessment Task Student Thinking.Â We then discuss mathematics classroom assessment and provide some specific examples of assessment strategies. The impact of large-scale assessment in mathematics education on curriculum, policy, instruction, and classroom assessment follows. We conclude by discussing the challenges that teachers face, as well as ways to support them. In each section, we pose problems worthy of continued or future research. Instructional quality can improve when professionals work together. It is evident that substantial professional learning for teachers will be needed to successfully implement the CA CCSSM.Â The MP standards represent a different vision of what students should be doing in classrooms. Students may investigate mathematical concepts with manipulatives for an entire class period or work on the same mathematics problem for a substantial amount of time.Â The support of college and university personnel for high-quality mathematics instruction is also crucial. Personnel from institutions of higher education support Kâ€“12 mathematics education by joining in partnership with their local schools.